

# Estimating DEA Confidence Intervals for Canadian Urban Paratransit Agencies Using Panel Data Analysis

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## Abstract

This paper illustrates three concepts new to the Data Envelopment Analysis (DEA) literature, and applies them to data from Canadian urban paratransit agencies. First, it predicts valid confidence intervals and trends for each agency's true efficiency. Second, it uses Panel Data Analysis methodology, a set of statistical procedures that are more likely to produce valid estimates than those commonly used in DEA studies. Third, it uses a new method of identifying and adjusting for environmental effects that has more power than conventional procedures.

## INTRODUCTION

Data Envelopment Analysis (DEA) has been increasingly utilized to compare urban transit agency performance. De Borger, Kerstens and Alvaro (2002) identify 15 articles published before 2001, and since then at least 15 more have been published or are in press (Boilé 2001, Nolan et al. 2001, Novaes 2001, Odeck and Alkadi 2001, Pina and Torres 2001, De Borger et al. 2002, Nolan et al. 2002, Karlaftis 2003, Boame 2004, Karlaftis 2004, Brons et al. 2005, Odeck 2006, Sheth et al. 2006, Graham 2006, Barnum et al. 2006).

The reason for DEA's popularity is easy to understand. In one of the earlier DEA studies of urban transit, Chu, Fielding and Lamar (1992, p. 224) argue that "performance analysis needs to progress from multiple measures and partial comparison to more robust indicators of performance . . . so that the achievements of one agency can be examined in reference to peer group agencies." More recently, Brons et al (2005, pp. 1-2) add "it is of great interest to investigate whether urban transit operators work in a technically efficient way (i.e. reach economic targets such as cost minimization or output maximization conditional on output or input constraints). Solid technical efficiency (TE) measurement can provide a significant contribution." DEA provides a single comparative measure of technical efficiency in analyses involving multiple inputs and multiple outputs, so it is uniquely equipped to fulfill the needed role.

One of DEA's major shortcomings is that it reports only a point estimate of the technical efficiency of a Decision Making Unit (DMU), with no indication of the interval within which the DMU's true technical efficiency is likely to occur (Boame 2004). A DEA score is composed of both true efficiency and statistical noise, but the entire score is treated as true efficiency. Without an estimate of the size and characteristics of the disturbance term, it is impossible to determine with statistical significance whether a given DMU is efficient (Grosskopf 1996).

Bootstrapping of cross-sectional data has been used to estimate the confidence interval within which a true production frontier occurs, with the results used to estimate the range within

which the DEA score of a fixed input-output set occurs (Xue and Harker 1999, Simar and Wilson 2004, Boame 2004). However, the reported confidence interval for the target DMU's efficiency is solely the result of variation of the production frontier, because the statistical noise in the target DMU's data is not taken into consideration. Boame (2004, p. 404) states it more elegantly: "the bootstrap efficiency estimate for the  $i^{\text{th}}$  firm is evaluated as the efficiency of the original input  $x_i$  relative to the bootstrapped isoquant of the input set." Thus, these bootstrapping procedures account for stochastic variations in the production frontier (the isoquant), but do not address stochastic variations in the inputs and outputs of the target DMU. So, they alone cannot estimate the range within which a target DMU's true efficiency occurs.

In order to estimate the interval within which a given DMU's true DEA efficiency occurs, it is necessary to utilize multiple observations of each DMU's efficiency scores, which, of course, is true for any stochastic variable. Cross sectional data have only one observation for each DMU, so they cannot be used to estimate these confidence intervals. If panel (longitudinal) data are available, but they are analyzed by cross-sectional methods or by pooling, then they also do not provide the multiple observations needed. To date, all urban transit DEA studies that have panel data have used cross-sectional methods or pooling (Chang and Kao 1992, Nolan 1996, Button and Costa 1999, Nolan et al. 2001, Nolan et al. 2002, Karlaftis 2003, Karlaftis 2004), and we are aware of only one DEA study with panel data that does not do so (Steinmann and Zweifel 2003).

In this paper, therefore, we use Panel Data Analysis (PDA), a unique set of statistical methods that are much more informative than the statistical techniques used in current DEA research (Wooldridge 2002, Hsiao 2003, Frees 2004, Baltagi 2005, Baum 2006). By simultaneously estimating the cross-sectional and longitudinal aspects of panel data, PDA

makes it possible to develop valid confidence intervals for individual DMUs, and also it improves the power, precision and validity of other statistical estimates.

Another important issue for DEA is the need to identify exogenous influences in order to explain variations in DEA scores caused by factors external to the DMUs, both for policy purposes and in order to correctly evaluate the endogenous efficiency of individual DMUs. Two-stage methods are the most popular procedures for identifying environmental influences.

With the conventional two-stage method, a DEA is first conducted using only traditional (endogenous) inputs and outputs. Then, the first-stage DEA scores are regressed on the environmental/contextual (exogenous) variables of interest. The regression outcomes are used to identify exogenous inputs that influence the first-stage DEA scores to a statistically significant degree, and to adjust DEA scores to account for these influences. Statistical significance tests usually are based on asymptotic theory, although some prefer bootstrapping (Kerstens 1996, Pina and Torres 2001, Ray 2004, Cooper et al. 2004, Boame 2004, Coelli et al. 2005, Simar and Wilson 2007).

While it has been long suspected (Grosskopf 1996, Coelli et al. 2005), it recently has been demonstrated that the conventional two-stage method exhibits substantial bias, low precision, and low power (Barnum and Gleason 2007). These results were based on asymptotic methods. However, bootstrapping also has been shown to have low power in detecting true relationships (Zelenyuk 2005).

A reverse two-stage procedure, that yields estimates without the bias, precision and power problems that compromise the validity of the conventional method's estimates, has been demonstrated with simulated data (Barnum and Gleason 2007). In essence, this procedure reverses the conventional method's steps. In the first stage, the endogenous (traditional) inputs of interest are regressed on the exogenous (environmental) factors expected to influence them. The resulting estimates are used to adjust the endogenous inputs to remove

the marginal influence of exogenous factors. In the second stage, the outputs and adjusted inputs are analyzed by DEA, with the resulting scores being independent of exogenous effects. (The same procedure could be applied to outputs, if they were influenced by exogenous factors.) Herein, we apply the reverse two stage method to real-world empirical data, using PDA instead of the conventional statistical methods.

With this paper, therefore, we illustrate three new techniques for DEA research. First, we provide a valid method for estimating confidence intervals for individual DMUs. Second, we apply Panel Data Analysis to all statistical procedures. Third, we illustrate an improved two-stage method for adjusting for environmental influences.

We apply these techniques to DEA efficiency measurement of urban paratransit operations, in itself an important topic that has not been much addressed previously. In the only paper of which we are aware, Viton (1997) included both conventional bus and demand-responsive urban transit in his DEA, but did not analyze them separately. As noted in a current Transit Cooperative Research Program project (2006), "Demand-response transportation (DRT) systems are under increasing pressure to improve performance because of increased demand for service and financial constraints. . . . To identify opportunities for improvement, DRT systems need better . . . methods to measure and assess performance." For this study, we use Canadian urban paratransit data because it is more detailed than U.S. data, and also because it will be only the second DEA study using Canadian transit data (Boame 2004).

#### **THE DATA SET, OUR ASSUMPTIONS, AND PREVIEW OF THE PAPER**

Data are from 28 Canadian urban paratransit agencies for 1996-2004. They include all agencies that reported the data needed for this analysis (Canadian Urban Transit Association, 1996-2004).

As is typical with traditional DEA, we do not suggest that these 28 DMUs are a random sample that is representative of all possible paratransit agencies at all possible times. The 28 DMUs represent the population and time period of interest, and those that are reported to be efficient define the true production frontier. That is, we adopt the concept that a DMU's relative efficiency is determined solely by comparisons with the other DMUs in the analysis (Charnes et al. 1978, Cooper et al. 2004). In contrast to traditional DEA, we address the fact that any DEA score is composed of both the true level of technical efficiency and a random error component.

Below, the inputs and outputs are identified and justified. The first stage Panel Data Analysis is used to adjust the endogenous inputs to remove the effects of exogenous (environmental) influences. The second stage DEA model is presented, followed by the statistical model used in the second stage Panel Data Analysis. The results of diagnostic tests on the second-stage statistical error terms are presented. Finally, for each DMU, the predicted values, confidence intervals and trends that resulted from the procedure are examined, followed by the results of the conventional two-stage method and our conclusions.

### **DEA INPUTS AND OUTPUTS**

Canadian urban paratransit agencies are the DMUs, with each using two inputs to produce four outputs. Inputs are (1) annual operating expenses of paratransit service dedicated to disabled individuals, and (2) annual operating expenses attributable to disabled riders of non-dedicated paratransit service. Outputs are (1) annual number of disabled passenger trips on paratransit service dedicated to disabled individuals, (2) annual number of disabled passenger trips on non-dedicated paratransit service, (3) annual operating revenue from dedicated paratransit service, and (4) annual operating revenue attributable to disabled riders of non-dedicated paratransit service. This particular set of inputs and outputs, and

organizational subunits, is based on those considered key by the industry and reflected in the

set of published performance indicators that are used to compare agencies (Canadian Urban Transit Association, 1996-2004).

Two key outputs of Canadian urban paratransit agencies are disabled passenger trips and operating revenue (mostly fares). Fare levels vary significantly among systems and years; consequently, passenger trip and revenue values measure two unique outputs. Agencies have different mixes of these two outputs, because increases in one lead to decreases in the other when fares are changed, holding inputs constant (Litman 2004). In setting fare levels, it would never be a behavioral objective of these agencies to minimize costs or to maximize either revenues or profits. Generally, fares are set at levels that local decision makers consider “fair,” and may be based on factors such as local farebox recovery ratio objectives and local mass transit fares. This will result in a variety of revenue-rider ratios, none being universally superior because of the absence of a common behavioral objective.

Both outputs are produced by two distinct organizational subunits: a subunit serving only disabled users (dedicated service), and a subunit serving both disabled and non-disabled users (non-dedicated service). Dedicated service is provided by vehicles exclusively dedicated to the transport of persons with disabilities, which may be operated by the agency itself or subcontracted. Non-dedicated service is provided by vehicles that serve both disabled and non-disabled customers, often taxicabs subcontracted by the paratransit agency to transport its disabled clients.

Operating expenses are used as a proxy for physical inputs. It is more typical in transit DEA studies to use physical inputs, most often labor, fuel and vehicles (De Borger et al. 2002). We have not done so for the following reasons. DEA assumes that there is substitutability among inputs, with diminishing marginal rates of substitution (Petersen 1990). For paratransit, this would mean that, to produce a fixed level of output, a DMU could

substitute labor for vehicles, or substitute vehicles for fuel. In truth, there is very little substitutability in this industry; inputs have to be used in a virtually fixed ratio, with any excess being wasted. Disaggregating non-substitutable inputs would usually result in some truly inefficient units being reported as efficient. As is well known, increasing the number of inputs and outputs results in higher efficiency scores, whether or not they are justified. Also, the number of DMUs reporting slacks normally will increase, making the radial technical efficiency scores less meaningful. Finally, because of the random error component in all variables, the larger the number of inputs and outputs, the more likely that a DMU's efficiency score will be increased by random chance. Therefore, more valid technical efficiency scores will be reported if we aggregate the types of input resources that are not substitutes, weighting them by their prices, which is a conventional economic solution (Färe et al. 1994).

For this analysis, all inputs and outputs are limited to those related to disabled passengers. However, because there are two quite unique methods of providing transportation (dedicated or non-dedicated service), and two different organizational subunits with independent production processes, inputs and outputs from each are entered as separate variables. This disaggregation is necessary to avoid bias in the reported DEA scores. Recent research demonstrates that, for inputs and outputs that are allocable and substitutable for each other, as ours are, disaggregated data must be used to avoid bias in efficiency scores (Tauer 2001, Färe and Zelenyuk 2002, Färe et al. 2004, Färe and Grosskopf 2004, Barnum and Gleason 2005a, Barnum and Gleason 2005b, Barnum and Gleason 2006a, Barnum and Gleason 2006b).

#### **THE FIRST STAGE STATISTICAL MODEL: ADJUSTING FOR EXOGENOUS FACTORS**

In this study, the exogenous variables of interest are those that influence the level of input resources needed to produce a given amount of output. In paratransit, production and consumption are identical because trips are demand-based (unlike conventional transit where

more output is produced than consumed). Therefore, when analyzing paratransit efficiency in converting inputs into outputs, we do not have to deal with produced vs. consumed outputs, and the potential for erroneous performance indicators (Gleason and Barnum 1978, Gleason and Barnum 1982).

### **Exogenous Influences on Urban Paratransit Efficiency**

A number of exogenous variables that might influence Canadian urban paratransit input levels were considered. These included (1) annual snowfall, (2) whether the dedicated service was operated by the agency itself, contracted to a non-profit agency, or contracted to a for-profit management services company, (3) input resource price differences, and (4) average length of a passenger trip.

We expected areas with higher annual snowfalls to have higher expenses per trip, because most expenses are time-based and such areas could be expected to have slower travel speeds. Furthermore, there might be increased time involved in getting disabled passengers into and out of vehicles. Annual snowfall data for each of the nine years were collected for each of the 28 communities involved.

Dedicated service is provided by one of three options: the paratransit agency itself, another non-profit agency under contract with the paratransit agency, or a for-profit management services company under contract with the paratransit agency. We did not make predictions on the direction of influence.

A very high percentage of paratransit operating expenses is for employee compensation. Although Canadian data are not available, for U.S. paratransit in 2004, 82 percent of operating expenses was for employee compensation, followed by 6 percent for fuel, 6 percent for insurance, 5 percent for materials and supplies, and 1 percent for utilities (Danchenko 2006). And, average compensation varies substantially among the 28 Canadian agencies,

with the 2004 operator top base wage rate ranging from \$14.82 to \$24.32 per hour. Typically, most other wage rates in the dedicated service subunit are indexed on the operator top base wage rate, which also can serve as a proxy for wage rate variation in the non-dedicated subunit because it reflects local labor market conditions. It is likely that other operating expenses also will show somewhat similar variation due to local cost differences. Because employee compensation is mainly a reflection of the local labor market, and therefore largely beyond the control of the local management, its level is one of the exogenous variables for which we must adjust. We expect higher base rates to result in higher expenses per trip.

Because of exogenous factors such as geography and population density, trips will be longer in some areas than in others, and average speed will differ. The net effect of these local environmental factors can be summarized by the average number of vehicle hours per passenger. We would expect DMUs providing longer trips to have higher expenses per trip.

### **Statistical Models and Outcomes**

Variables that have no statistically significant influence are annual snowfall, and whether the dedicated service was provided by the paratransit agency, another non-profit agency, or a for-profit management services company. We don't present that model herein. The variables that do have a statistically significant influence are operator top base wage rate and vehicle hours per passenger:

$$\log(DOE_{jt}) = \alpha_j + \beta_1 \log(DedPass_{jt}) + \beta_2 \log(Wage_{jt}) + \beta_3 \log(VHPP_{jt}) + u_{jt} \quad (1)$$

$$\log(NOE_{jt}) = \alpha_j + \beta_1 \log(NDedPass_{jt}) + \beta_2 \log(Wage_{jt}) + \beta_3 \log(VHPP_{jt}) + u_{jt} \quad (2)$$

The subscript  $j = 1, \dots, 28$  represents the DMU involved and the subscript  $t = 1, \dots, 9$  represents the year. *DOE* is operating expenses for dedicated service, and *NOE* is operating expenses for non-dedicated service. *DedPass* is the number of passengers carried by the

dedicated subunit and  $NDedPass$  is the number of disabled passengers carried by the non-dedicated subunit.  $Wage$  is the operator top base rate in the dedicated subunit, and  $VHPP$  is the number of vehicle hours per passenger in the dedicated subunit. Because we have  $Wage$  and  $VHPP$  data only for the dedicated subunit, in the non-dedicated subunit they serve as proxies for the relative price of resources and the relative length of time traveled for the community involved. Finally,  $\alpha_j$  is the time-constant effect that reflects the unique characteristics of DMU  $j$ , and  $u_{jt}$  is the error term for DMU  $j$  in year  $t$ .

In order to correctly specify the models, it is necessary to include the number of passengers carried as an independent variable. Clearly, the main driver of operating expenses is passenger trips. If they were not included, some of their effect might be attributed to either wages or vehicle hours per passenger, and a host of other problems might result from an incorrectly specified model.

To estimate the parameters of these models we use PDA, treating the individual effect ( $\alpha_j$ ) as a fixed effect, with robust variance-covariance estimation in order to adjust for arbitrary serial correlation and heteroskedasticity. We used a fixed effect model in order to allow arbitrary correlation between the three observed explanatory variables and the individual effect  $\alpha_j$ . The residuals of the two equations were not correlated with each other [ $R^2 = 0.007$ ,  $F(1, 175) = 0.13$ ,  $P(F(1,175) > 0.13 = 0.7208)$ ], so we did not estimate the two equations as a set using Zellner's Seemingly Unrelated Regressions (SUR) model (Wooldridge 2002). All statistical analysis was conducted with Stata 9.2 (StataCorp 2007). The results are presented in Tables 1 and 2.

We use the expected values of the regression coefficients for wages and vehicle hours per passenger to remove the effects of these variables from the value of operating expenses for both subunits, namely

$$\text{Adjusted } \log(DOE_{jt}) = \log(DOE_{jt}) - 1.4991 \log(Wage_{jt}) - 0.3050 \log(VHPP_{jt}) \quad (3)$$

$$\text{Adjusted } \log(NOE_{jt}) = \log(NOE_{jt}) - 1.2656 \log(Wage_{jt}) - 0.5081 \log(VHPP_{jt}) \quad (4)$$

Note that these adjusted values retain any inefficiency contained in each DMU's individual intercept ( $\alpha_j$ ), as well as retaining its residual error ( $u_{jt}$ ). Only the expected effects of the exogenous variables are removed. The two adjusted input values and four original outputs are used in DEA.

#### THE SECOND STAGE DEA MODEL

For each of the nine years of data, DEA scores are computed using linear program (5) that is output oriented and reflects the constant returns to scale of this industry. The DEAs were conducted with Scheel's EMS software (2000). For each of the  $j$  DMUs ( $j = 1, \dots, 28$ ), there are data on the  $n = 2$  inputs  $x$  ( $x_{11}, \dots, x_{jn}$ ), and on the  $m = 4$  outputs  $y$  ( $y_{11}, \dots, y_{jm}$ ). The DEA score  $\theta$  identifies the technical super-efficiency of the target DMU  $k$  (Andersen and Petersen 1993).

$$\begin{aligned} & \max_{\lambda} \theta \\ & \text{subject to} \\ & \sum_{j=1}^{28} y_{jm} \lambda_j \geq \theta y_{km} \quad m = 1, 2, 3, 4 \\ & \sum_{j=1}^{28} x_{jn} \lambda_j \leq x_{kn} \quad n = 1, 2 \\ & \lambda_k = 0 \end{aligned}$$

$$\lambda_j \geq 0 \quad j = 1, 2, \dots, 28; j \neq k \quad (5)$$

The technical efficiency rather than the technical super-efficiency indicator would have been appropriate if our interest had focused on identifying the production frontier, those DMUs defining it, and the distance of inefficient DMUs from it. In that model, inefficient DMUs are compared to their efficient peers, but efficient DMUs are compared only to themselves. We are interested in always comparing the performance of each DMU to its efficient peers, whether or not it contributes to defining the production frontier. Super-efficiency scores provide the necessary comparison, because they always compare each DMU to its efficient peers regardless of its own efficiency level.

A second reason for using super-efficiency scores is to avoid a censored dependent variable in second-stage regressions (Coelli et al. 2005). Conventional technical efficiency scores yield a censored variable, because an efficient DMU's score of 1 will remain unchanged even if it were to become more productive by increasing outputs or decreasing inputs. Super-efficiency scores are an observable proxy for latent variable values underlying conventional efficiency scores, and they remove the need to estimate latent values using Tobit, sample-selected, or truncated regression (Breen 1996).

#### **THE SECOND STAGE STATISTICAL MODEL**

We use Panel Data Analysis with the fixed effects model because we wish to estimate the efficiencies of the specific 28 DMUs in the study, and the inferences are restricted to developing confidence intervals for the efficiency of each of these specific DMUs when compared to the remaining DMUs in the set (Baltagi 2005). The statistical model is

$$w_j \theta_{jt} = \alpha_j + \beta_j(t-1) + u_{jt} \quad j = 1, \dots, 28; t = 1, \dots, 9 \quad (6)$$

$\theta_{jt}$  is the super-efficiency score of DMU  $j$  in year  $t$ ,  $\alpha_j$  is the individual effect of DMU  $j$ ,  $\beta_j$  is the annual change in the individual effect  $\alpha_j$  of DMU  $j$ , and  $u_{jt}$  is the random error in the super-efficiency score ( $\theta_{jt}$ ) of DMU  $j$  in year  $t$ . Many DMUs showed linear trends in efficiency over the nine year period, so equation 6 includes a factor  $(t-1)$  that adjusts efficiency for the year involved, which permits a heterogeneous trend in each DMU's efficiency over the nine-year period. No exogenous variables are included in the model because their effects already have been removed via the first stage statistical model.

Not surprisingly, the unweighted scores resulted in heteroskedastic error terms among the DMUs. Because we develop confidence intervals for each DMU's true efficiency, we want the estimates to be statistically efficient as well as consistent. Therefore, each DMU's scores were weighted by  $w_j$ , an index based on the standard deviation of its errors when the unweighted scores were used.

After weighting the dependent variable, the error terms were homoskedastic; the Breusch-Pagan/Cook-Weisberg test for heteroskedasticity found no statistically significant differences at the 0.05 level among the DMUs [ $\chi^2(27) = 0.03$ ,  $P(\chi^2 > 0.03) = 1.0000$ ], over time [ $\chi^2(1) = 0.01$ ,  $P(\chi^2 > 0.01) = 0.9383$ ], or by expected value of the response variable [ $\chi^2(1) = 0.02$ ,  $P(\chi^2 > 0.02) = 0.8965$ ].

#### **ERROR TERM DIAGNOSTICS**

For confidence intervals from a correctly specified regression model, residuals usually are assumed to be independent and identically distributed (i.i.d.), and Normally distributed. When the response variable is a DEA score, these characteristics cannot be taken for granted. The main reason for this is because each DMU's score is influenced by the performance of other

DMUs. If the same DMUs consistently influence each other, it may cause correlations among their error terms. Such contemporaneous correlation would invalidate the requirement for independent residuals (Xue and Harker 1999, Simar and Wilson 2007).

Although contemporaneous correlation does not bias the expected value of estimated efficiency levels, variance estimates can be more precise if the correlation is taken into account. That is, if the residuals are contemporaneously correlated to a statistically significant degree, one could use the estimates in a Generalized Least Squares model to decrease standard errors.

Tests for contemporaneous correlation cannot be conducted if the data are cross-sectional, or if cross-sectional methods are used on panel data. Using PDA, however, such tests are available. If the number of DMUs exceeds the number of time periods, as is true for our data, then tests for cross-sectional independence of residuals include Freedman's  $R_{AVE}$  and Frees'  $R_{AVE}^2$  evaluated with his Q-distribution (Frees 1995, Frees 2004), Pesaran's  $CD$  cross-sectional dependence test (Baum 2006, p. 222), and the pairwise correlation of residuals.

The results are  $CD = 0.073$ ,  $P( CD > |0.073| ) = 0.9422$ ; and  $R_{AVE} = 8.438$ ,  $P( R_{AVE} > 8.438 ) = 0.9998$ . None of the pair-wise correlations were statistically significant at the 0.05 level, using either Bonferroni-adjusted significance tests or Sidak-adjusted significance tests. However,  $R_{AVE}^2 = 1.392$ , which is statistically significant at the 0.05 level.

Because only three of the four tests reported no statistically significant contemporaneous correlation, the evidence was not unanimous. The conservative decision would be to assume that there is no cross-correlation. This decision permits the use of more robust statistical

models, which report the same mean values but wider variances. Therefore we do not correct for possible cross-correlation.

There are other conditions violating i.i.d and Normal distribution assumptions that cannot be identified with conventional methods, but can be tested for (and corrected) with PDA. These include heteroskedasticity across DMUs, the Normal distribution of residuals, and serial correlation.

As discussed earlier, the Breusch-Pagan/Cook-Weisberg test for heteroskedasticity found no statistically significant differences among the DMUs. The Shapiro-Wilk  $W$  test did not reject the hypothesis of Normality [ $z = 0.599$ ,  $P(z > 0.599) = 0.275$ ]. To test for first-order serial correlation, we regressed each residual on the prior year's residual using all 80 of the available observations. Because the fixed effect model demeans the data, if the true error terms are uncorrelated then the estimated errors will have a correlation coefficient of  $-1/(T-1)$ , with  $T$  equal to the number of observations for each DMU (Wooldridge 2002). Because  $T = 5$ ,  $\rho = -0.25$  if there is no serial correlation. Here,  $\hat{\rho} = -0.3349$ , with a standard error of 0.1070, which is not different from -0.25 to a statistically significant degree.

In sum, the null hypotheses that the residuals are i.i.d. and Normally distributed cannot be rejected. This demonstrates that violations of the standard parametric assumptions by DEA residuals are not inevitable. It may be worthwhile to recall that any disturbance term encapsulates complicated, unidentified interactions with the variables in question, so it is never truly random. But, a residual can be treated as random if it meets appropriate statistical tests for randomness (Frees 2004). So, we make the standard asymptotic assumptions in developing confidence intervals from equation 6.

## **RESULTS FOR THE REVERSE TWO-STAGE PROCEDURE**

The regression based on equation 6 had an R-square of 0.9837 [ $F(55, 196) = 215.10$ ,  $P(F(55,196) > 215.10) < 0.00005$ ]. Because the equation permits heterogeneous trends in

each DMU's efficiency, we tested to assure that trends indeed were present. Using the Chow test to compare the full model with a reduced model in which the trend variables are removed, the difference was statistically significant at the 0.05 level [ $F(28, 196) = 5.90$ ,  $P(F(28, 196) > 5.90) < 0.00005$ ]. Therefore, for at least some of the DMUs, there are statistically significant trends in their efficiency levels over the nine year period.

The DEA model is output oriented, so higher scores mean lower efficiency. The confidence intervals are based on the standard error of prediction of the true mean value, at the 0.90 level of confidence. These scores have been “unweighted” in Table 3, so they are normal super-efficiency measures.

As shown in Table 3, the 2004 point estimates of the mean efficiency for 16 DMUs showed them to be inefficient, but 6 of these were not inefficient to a statistically significant degree. Of the 12 DMUs with efficient mean point estimates, 8 were not efficient to a statistically significant degree. Thus, whether 14 of the 28 systems were or were not efficient in 2004 cannot be determined with statistical confidence.

Therefore, the point estimates of the expected efficiency for a given year should not be used to positively identify their levels of efficiency, because the range within which their true efficiency could occur is often going to overlap the inefficient and efficient ranges. Indeed, because the actual DEA scores for 2004 have a much wider variation than the estimated mean scores, attempting to determine a DMU's true efficiency from those single data points would be even more unwise.

Whether each DMU's trend in efficiency is statistically significant also is of interest.  $\hat{\beta}_j < 0$  indicates efficiency is increasing, and  $\hat{\beta}_j > 0$  indicates efficiency is decreasing. For the DMUs, 18 report increasing efficiency over the nine years involved, of which 12 show

statistically significant improvements. On the other hand, 10 report decreasing efficiency over the nine year period, of which 5 are worse to a statistically significant degree.

## RESULTS FOR THE CONVENTIONAL TWO-STAGE PROCEDURE

It is of interest to compare the results of our reverse two-stage procedure used above with the results of the conventional two-stage procedure. Under the conventional two-stage procedure, first DEA scores are calculated from the unadjusted, endogenous inputs and outputs, and second the unadjusted scores are regressed on the exogenous variables. We did so, using linear program 5 for the DEA and statistical model 7 for the regression. Model 7 is identical to model 6, except that the two exogenous factors are included as independent variables. Those applying the conventional procedure make the standard i.i.d. and Normality assumptions, but cannot verify them because of their use of cross-sectional methods, so we don't verify those assumptions here.

$$\theta_{jt} = \lambda_1(VHPP_{jt}) + \lambda_2(Wage_{jt}) + \alpha_j + \beta_j(t-1) + u_{jt} \quad j = 1, \dots, 28; t = 1, \dots, 9 \quad (7)$$

The data cannot validly be pooled, because the nine error terms of each DMU (one for each of the nine years) are not independent when the data are pooled; indeed, when the residuals from the pooled data are regressed on their own residuals lagged by one year, the regression coefficient is 0.82,  $t = 22.4$  ( $P > |t| < 0.00005$ ), and  $R^2 = 0.69$ . Further, comparing the full model in equation 7 with a reduced model that includes only the *VHPP* and *Wage* variables, the full model explains 90 percent of the variance in efficiency scores, as compared to only 26 percent for the reduced model; these values differ to a statistically significant degree [ $F(57,201) = 50.11$ ,  $P(F(57,201) > 50.11 < 0.00005)$ ], so the reduced model is misspecified. Any pooling of this data, or the use of any one-year cross section, would yield invalid estimates and confidence intervals.

For the regression using equation 7, the regression coefficient of *VHPP* is -0.236, which is in the right direction, but its t-ratio is only 0.91, not significant at the 0.37 level. The regression coefficient for *Wage* is -0.236, which is in the right direction, but it has a t-ratio of only -0.68 so it is not significant at the 0.50 level. In short, neither exogenous variable is reported to affect the DEA score to a statistically significant degree. This is not surprising because both are significantly correlated with endogenous inputs, as shown in Tables 1 and 2; Barnum and Gleason (2007) have shown that the power of the conventional two-stage procedure to detect the effects of exogenous factors rapidly decreases as correlation between the exogenous variables and endogenous inputs increases.

## **CONCLUSIONS**

As exhibited in this paper, Panel Data Analysis provides a methodology for estimating valid confidence intervals for the DEA efficiency of individual DMUs. It addresses noise in the data of the target DMU and the production frontier, and it makes it possible to test residuals before accepting (or rejecting) the standard asymptotic assumptions. By exploiting the advantages of simultaneous estimation of the cross-sectional and longitudinal aspects of panel data, it increases the validity of all parameter estimations, and permits identification of statistically significant trends.

Further, the paper demonstrates a new method of identifying and adjusting for the effects of environmental variables on efficiency, herein called the reverse two-stage method. The reverse two-stage method is shown to have more power to detect environmental effects than the conventional two-stage procedure, at least in situations where the endogenous inputs and exogenous influences are correlated, and it avoids some of the other well-known statistical shortcomings of the conventional method.

**Table 1. Fixed-effects Robust Regression of the Log of Dedicated Subunit Operating Expenses on Selected Independent Variables and Individual Effects**

$\log(DOE_{jt})$	Coefficient	Robust Standard Error	t-ratio	$P(t)> t $
$\log(DedPass_{jt})$	0.6905	0.0622	11.09	0.000
$\log(Wage_{jt})$	1.4991	0.0833	18.00	0.000
$\log(VHPP_{jt})$	0.3050	0.0810	3.77	0.000
constant	2.350	0.6270	3.75	0.000
$\hat{\sigma}_\alpha$	0.5404			
$\hat{\sigma}_u$	0.1041			
$R^2_{\hat{\alpha},DOE}$	0.9642			
$R^2$ within	$R^2$ between	$R^2$ overall	$F(3, 221)$	$Prob(F >$
= 0.7426	= 0.9744	= 0.9680	= 52.96	252.96) =
				0.0000

**Table 2. Fixed-effects Robust Regression of the Log of Non-Dedicated Subunit Operating Expenses on Selected Independent Variables and Individual Effects**

$\log(NOE_{jt})$	Coefficient	Robust Standard Error	t-ratio	$P(t) >  t $
$\log(NDedPass_{jt})$	0.9220	0.0598	15.41	0.000
$\log(Wage_{jt})$	1.2656	0.2457	5.15	0.000
$\log(VHPP_{jt})$	0.5081	0.1668	3.05	0.003
constant	0.0016	0.6929	0.00	0.998
$\hat{\sigma}_\alpha$	0.3286			
$\hat{\sigma}_u$	0.2414			
$R^2_{\hat{\alpha}, NOE}$	0.6496			
$R^2$ within	$R^2$ between	$R^2$ overall	F(3,153)	Prob(F >
= 0.7657	= 0.9711	= 0.9615	= 146.89	146.89) =
				0.0000

**Table 3. 2004 Super-efficiency Scores, 28 Canadian Urban Paratransit Properties**

$DMU_j$	2004 Super-efficiency Score, Output Oriented (0.90 Confidence Interval)			Conclusion	Mean Annual Change ( $\hat{\beta}_j$ )
	$E(\theta_{j9})$	Lower limit	Upper limit		
1	1.003	0.818	1.188	Inefficient	0.017
2	0.925	0.760	1.090	Efficient	-0.008
3	1.291	1.186	1.396	Inefficient*	-0.036*
4	0.735	0.610	0.861	Efficient*	0.062*
5	1.036	0.951	1.121	Inefficient	-0.007
6	0.955	0.742	1.167	Efficient	-0.06*
7	0.992	0.753	1.232	Efficient	-0.083*
8	0.867	0.727	1.007	Efficient	-0.034*
9	1.282	1.189	1.374	Inefficient*	0.001
10	1.182	1.010	1.355	Inefficient*	-0.035
11	1.043	0.838	1.249	Inefficient	-0.05*
12	0.750	0.503	0.998	Efficient*	-0.048
13	1.530	1.435	1.624	Inefficient*	-0.012
14	1.282	1.212	1.352	Inefficient*	0.029*
15	0.868	0.772	0.965	Efficient*	0.002
16	0.853	0.810	0.895	Efficient*	-0.009
17	1.089	0.954	1.223	Inefficient	-0.066*
18	1.108	0.972	1.245	Inefficient	0.019
19	0.968	0.907	1.029	Efficient	0.002
20	0.989	0.867	1.111	Efficient	-0.064*
21	2.090	1.949	2.232	Inefficient*	0.041*
22	0.874	0.702	1.047	Efficient	-0.104*
23	1.235	1.145	1.324	Inefficient*	-0.03*
24	1.448	1.406	1.490	Inefficient*	-0.019*
25	1.086	1.018	1.154	Inefficient*	-0.017*
26	0.852	0.666	1.038	Efficient	0.044*
27	1.030	0.940	1.120	Inefficient	-0.042*
28	1.288	1.134	1.442	Inefficient*	0.036*

\* Statistically significant at the 0.10 two-tailed level.

Note:  $E(\theta_{j9}) \leq 1$  is efficient,  $E(\theta_{j9}) > 1$  is inefficient,  $\hat{\beta}_j < 0$  indicates the DMU is becoming more efficient, and  $\hat{\beta}_j > 0$  indicates the DMU is becoming less efficient.

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