# Estimating Data Envelopment Analysis Frontiers for Nonsubstitutable Inputs and Outputs: The Case of Urban Mass Transit

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### Estimating Data Envelopment Analysis Frontiers for Nonsubstitutable Inputs and Outputs: The Case of Urban Mass Transit

### Abstract

Conventional data envelopment analysis (DEA) models assume that inputs are substitutable for each other, and that outputs are substitutable for each other. However, recent DEA articles frequently include outputs that cannot be substituted for each other and inputs that cannot be substituted for each other. In this paper, we demonstrate that conventional DEA models report invalid efficiency scores when outputs and/or inputs are nonsubstitutable. We use artificial data to illustrate the differences between the efficient frontiers of substitutable and nonsubstitutable variables. Assuming that the inputs and outputs are nonsubstitutable, we compare the DEA scores from a conventional DEA model with those from a new model, the Fixed Proportion Additive (FPA) model, which we developed to deal with nonsubstitutable variables. Then, we apply the conventional and FPA models to real-world data involving urban mass transit systems, where the outputs are nonsubstitutable, and where the inputs are nonsubstitutable. Finally, we make recommendations for model use when inputs or outputs are nonsubstitutable, one involving the development of new models and the others involving adaptations that can be made if one wishes to use conventional models.

### INTRODUCTION

The requirements for input substitutability and output substitutability have been maintained in all data envelopment analysis (DEA) models of which we are aware (Banker and Maindiratta, 1986; Petersen, 1990; Färe, Grosskopf and Lovell, 1994; Bogetoft, 1996; Chang, 1999; Kuosmanen, 2001; Takeda and Nishino, 2001; Post, Cherchye and Kuosmanen, 2002; Färe and Grosskopf, 2004; Coelli et al., 2005; Podinovski, 2005; Cooper, Seiford and Tone, 2007).

In this paper, we consider situations in which inputs or outputs are nonsubstitutable. Of the 18 non-agricultural DEA papers published or forthcoming in Applied Economics in 2006 and 2007, only 3 use substitutable variables exclusively. That is, there are at least two nonsubstitutable inputs or outputs in 15 of the 18 papers. Thus, nonsubstitutable variables are common in DEA studies.

After presenting definitions and assumptions, we identify examples of nonsubstitutable inputs and outputs from recent DEA articles, illustrate the consequences of nonsubstitutability, compare substitutable and nonsubstitutable DEA scores using real-world data from urban mass transit systems, and suggest valid models for dealing with nonsubstitutable sets of variables.

### **DEFINITIONS AND ASSUMPTIONS**

*Substitutable inputs* can be used interchangeably for the production of a fixed amount of output; if an efficient decision making unit (DMU) uses less of one input, then it must use more of the other inputs if all outputs are to remain constant. Conventional DEA and Free Disposal Hull (FDH) models both assume that inputs are substitutable (Tulkens, 1993; Färe, Grosskopf and Lovell, 1994; Thrall, 1999; Coelli *et al.,* 2005; Cooper, Seiford and Tone, 2007).

A fixed amount of input can be allocated in different proportions to **substitutable outputs**; if an efficient DMU produces more of one output, then it must produce less of the other **UIC Great Cities Institute** 1 outputs if all inputs are to remain constant. Both DEA and FDH rely on the assumption that outputs are substitutable (Tulkens, 1993; Färe, Grosskopf and Lovell, 1994; Thrall, 1999; Coelli *et al.*, 2005; Cooper, Seiford and Tone, 2007).

*Nonsubstitutable inputs* cannot replace each other in the production of a fixed amount of output; that is, there is no factor substitutability. Most commonly, the inputs must be utilized in a fixed ratio to produce the output of choice; any quantity of an input in excess of the ratio required is wasted. Such production technologies usually are referred to as Leontief or fixed factor proportion technologies (Beattie and Taylor, 1985; Barnum and Gleason, 2006). The ratio may vary across internally efficient DMUs due to environmental influences, but it never would be possible for an efficient DMU to maintain constant output by increasing one nonsubstitutable input and decreasing another.

If outputs are **nonsubstitutable**, an efficient DMU cannot change the amount of each that is produced by adjusting the proportion of a fixed input that each receives; that is, for a given amount of input the product proportions are fixed. Outputs can be nonsubstitutable because their production is inseparable (such as mutton and wool); because the amount of one output determines the amount of the other output; because the input being used is not allocable between the particular set of outputs involved; or because the input is inherently nonallocable (Beattie and Taylor, 1985; Barnum and Gleason, 2006). As with nonsubstitutable inputs, the ratio of nonsubstitutable outputs may vary across internally efficient DMUs due to environmental influences. But, it never would be possible for an efficient DMU to increase one nonsubstitutable output and decrease another by reallocating the input.

The requirement that *each variable must be homogeneous* is a ubiquitous assumption for DEA (Charnes, Cooper and Rhodes, 1981; Dyson *et al.*, 2001) and, indeed, for all valid economic analyses (Beattie and Taylor, 1985). That is, the *relevant* characteristics and

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environment of a given type of input (output) are assumed to be identical in all instances involving that input (output). In order to validly classify variables as substitutable or nonsubstitutable, they must be sufficiently homogeneous so that any remaining differences do not affect their substitutability.

### EXAMPLES OF NONSUBSTITUTABLE INPUTS AND OUTPUTS

Our examples of nonsubstitutable inputs and outputs are drawn from 2006-2007 articles in *Applied Economics*. We offer examples of variables that normally would be nonsubstitutable, and which would require empirical testing to confirm substitutability if they are to be treated as substitutable. We illustrate such testing for our real-world examples later in the paper.

The most common industry analyzed is banking, and the papers provide many examples of nonsubstitutable inputs and outputs. A few examples of bank inputs that normally would be nonsubstitutable include deposits vs. interest expenses (Guzmán and Reverte, 2007); deposits vs. salaries (Liu and Lin, 2007; Liu, 2007a); number of employees vs. number of branches (Damar, 2006); and expenses vs. number of employees (Damar, 2006). Some of the bank outputs that normally would be nonsubstitutable include loans vs. interest income (Guzmán and Reverte, 2007); and deposits vs. loans (Damar, 2006).

Other examples of nonsubstitutable inputs include number of employees vs. materials expenditures in construction (You and Zi, 2007); acreage of container storage vs. total length of container berths in ports (Liu, 2007b); boat length vs. engine power in fishing vessels (Pascoe, 2007); number of vehicles vs. gallons of fuel in garbage collection (Liu, 2006); number of workers vs. line length in electricity distribution (von Hirschhausen, Cullmann and Kappeler, 2006); and number of players vs. total wages in football (soccer) teams (Barros and Leach, 2006).

Additional examples of nonsubstitutable outputs include households with water vs. households with sewage disposal provided by local governments (Sung, 2007); water delivered vs. population supplied for water utilities (García-Valiñas and Muñiz, 2007); number of port calls vs. cargo handled in ports (Liu, 2007b); number of car-trips vs. car personnel in garbage collection (Liu, 2006); number of customers vs. units of electricity sold in electric distribution (von Hirschhausen, Cullmann and Kappeler, 2006); points scored vs. ticket sales in football teams (Barros and Leach, 2006); and life expectancy vs. GDP per capita in healthcare (Grosskopf, Self and Zaim, 2006).

Certainly, there may be cases where some of the preceding inputs and outputs are substitutable, but for the typical case with homogeneous variables, it appears to us that most would be nonsubstitutable most of the time. Therefore, it is important to identify the effects of nonsubstitutability on DEA efficiency measurement. We start by examining inputs.

### INPUTS

In DEA, the production frontier for two substitutable inputs producing a fixed amount of output is estimated deterministically based on those DMUs that use the least of one input for a given level of the other input, and production possibilities are anywhere above and to the right of that frontier, as shown by the dotted line in Figure 1. The only way that a DMU on the frontier can decrease its use of one of the inputs is to increase its use of the other input. The underlying assumption is that the inputs are substitutes; if less of one input is used to produce a fixed amount of product, then more of the other must be used (Beattie and Taylor, 1985). The substitutability requirement is necessary to justify the estimate of a convex, piecewise frontier over the data, where the slope of each piece is greater than the last as one moves toward the origin along the horizontal axis.

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When two inputs are nonsubstitutable (that is, they must be used in a fixed ratio by efficient DMUs), then the production frontier is right-angled and convex toward the origin (Beattie and Taylor, 1985). There is no economic requirement for a DMU that is efficient in the use of one input to be efficient in the use of the other. As a consequence, the vertical frontier can be estimated deterministically by the DMU that uses the least of Input 1, and the horizontal frontier can be estimated deterministically by the DMU that uses the least of Input 2. As is shown in Figure 1 by the dashed lines, a DMU that is efficient in the use of both inputs must produce at the point that represents the minimum possible amount of each input individually, namely 6 units of Input 1 and 6 units of Input 2. In our example, no DMU is completely efficient; if one were efficient in the use of both inputs, then it would consume six units of each.





Interestingly enough, as we show later with real-world data, even when it has been proven conclusively that inputs are nonsubstitutable, random variation and differences in the efficiency with which each input is used by DMUs result in a deterministic frontier that appears to represent substitutable inputs.

### Measuring Input Efficiency

In order to estimate each DMU's efficiency relative to the right-angle frontier in Figure 1, one simple possibility would be a variation on the additive model (ADD) (Charnes *et al.*, 1994). We call the variation the Fixed Proportion Additive (FPA) model, because it assumes that the production frontiers are at right angles. Using the FPA model, the rectilinear distance between a DMU and the efficient point is a measure of the DMU's inefficiency.

In order to estimate each DMU's efficiency relative to the conventional frontier in Figure 1, we use the ADD model. Recall that, with the ADD model, the efficient point for a DMU is the furthest point on the frontier where neither of its inputs has increased and its output has not decreased. The rectilinear distance between the DMU and the efficient point is a measure of the DMU's inefficiency (Charnes *et al.*, 1994). Table 1 exhibits the reported efficiencies for the six DMUs using the FPA model and the ADD model.

As can be seen, the ADD model reports that the first five DMUs are efficient while the FPA model reports none of the DMUs are efficient, as would be expected based on their respective frontiers. That is, if the inputs are nonsubstitutable, the conventional model falsely classifies all but one of the inefficient DMUs as being efficient.

Table 1.	Compariso	n of ADD a	and FPA So	cores for In	puts, Artificial Data
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DMU	Input 1	Input 2	Score	Score		
	input i	input 2	ADD	FPA		
A	6	80	0	74		
В	8	60	0	56		
С	24	20	0	32		
D	50	10	0	48		
Е	90	6	0	84		
F	30	30	16	48		

Note: Lower scores represent higher efficiency; efficient DMUs will score zero. The ADD model is the conventional CRS, input-oriented additive model. The FPA model is the Fixed Proportion Additive model, with efficient input being 6 units of Input 1 and 6 units of Input 2.

Moreover, because inefficient DMU F's ratio of inputs (30/30) is identical to the ratio of the single point of efficient production (6/6) under nonsubstitutability, we can compare the radial efficiencies of DMU F under the two assumptions. Under the nonsubstitutable inputs assumption, its radial efficiency is 20 percent; under the substitutable inputs assumption, its radial efficiency is 76 percent. If the inputs are truly nonsubstitutable, then the radial, substitutable inputs model overstates the DMU's efficiency by 280 percent.

### OUTPUTS

In DEA, the production frontier for two substitutable outputs produced by a fixed amount of input is estimated deterministically based on those DMUs which produce the most of one output for a given level of the other output, and production possibilities are anywhere below and to the left of that frontier, as shown by the dotted line in Figure 2. The only way that a DMU on the frontier can increase its production of one of the outputs is to decrease its production of the other output. The underlying assumption is that the input is allocable, so if more of the input is

allocated to one product, than less is available for allocation to the other (Beattie and Taylor, 1985). For an efficient producer with fixed inputs, the outputs are substitutes because producing more of one requires producing less of the other. The substitutability requirement is necessary to justify the estimate of a concave, piecewise frontier over the data, where the slope of each piece is less than the last as one moves toward the origin along the horizontal axis. When two outputs are nonsubstitutable (that is, they must be produced in a fixed ratio by efficient DMUs), then the production frontier is right-angled and concave toward the origin (Beattie and Taylor, 1985). There is no economic requirement that a DMU efficient in the production of one output be efficient in production of the other. As a consequence, the vertical frontier is estimated deterministically by the DMU that produces the most of Output 1, and the horizontal frontier is estimated deterministically by the DMU that produces the most of output 2. As can be seen by the dashed lines in Figure 2, a DMU that is efficient in the production of both outputs must produce at the point that represents the maximum possible amount of each output individually, in this example 54 units of Output 1 and 60 units of Output 2.

Figure 2. Production Possibility Frontiers



### Measuring Output Efficiency

As with inputs, we estimate the efficiency of each DMU relative to the conventional frontier using the ADD model and each DMU's efficiency relative to the right-angle frontier using the FPA model. Table 2 exhibits the resulting efficiency data.

The conventional ADD model reports that the first five DMU are efficient, because it assumes that increasing the amount of one product can be achieved by decreasing the amount of the other, which, if the products are nonsubstitutable, clearly is not a valid assumption. If the products are nonsubstitutable, then the FPA model produces more valid efficiency estimates. In this case, the ADD model shows the first five DMUs to be efficient, while the FPA model shows UIC Great Cities Institute

those DMUs to all be inefficient and to vary greatly in their inefficiency levels. DMU F is inefficient under both models, but, not surprisingly, it is more inefficient with the FPA model.

			Score		
DMU	Output 1	Output 2	ADD	FPA	
А	5	60	0	49	
В	20	58	0	36	
С	40	50	0	24	
D	50	30	0	34	
Е	54	6	0	54	
F	27	30	33	57	

F27303357Note: Lower scores represent higher efficiency;<br/>efficient DMUs will score zero. The ADD model is the<br/>conventional CRS, output-oriented additive model.<br/>The FPA model is the Fixed Proportion Additive<br/>model, with efficient output being 54 units of Output 1

and 60 units of Output 2.

The output values of DMU F were chosen so their ratio (27/30) would be identical to that of the single efficient point (54/60) when fixed ratio production applies. So, we can compare the radial efficiencies of DMU F reported by the two assumptions. Assuming nonsubstitutability, its radial efficiency is 50 percent; and, assuming substitutability, its radial efficiency is 65 percent. If the nonsubstitutability assumption is correct, this means that DMU F's efficiency is overstated by 30 percent when nonsubstitutable outputs are incorrectly assumed to be substitutable.

## Table 2. Comparison of ADD and FPA Scores for Outputs, Artificial Data

### **APPLICATION TO URBAN MASS TRANSIT SYSTEMS**

### Statistical Testing

In this section, we illustrate statistical tests for determining the substitutability of inputs and outputs. The nature of the empirical relationship between any two variables of interest, holding everything else constant, provides evidence about their true substitutability. If two inputs (outputs) are substitutes then, holding other outputs and inputs constant, they will be negatively related. If two inputs (outputs) are not substitutable, then, holding other outputs and inputs constant, they will be positively related.

If there is no statistically significant relationship between two inputs (outputs), this might indicate that there is not a wide enough range of values or efficiencies to identify a relationship. In addition to looking for evidence of this possibility, we suggest that the researcher reconsider the choices of inputs and outputs, and perhaps apply more complex statistical models and more powerful statistical tests.

### Application Involving Nonsubstitutable Outputs

Two outputs sometimes used jointly in transit DEA studies are vehicle miles and passenger miles (De Borger, Kerstens and Costa, 2002; Karlaftis, 2004; Odeck, 2006). We hypothesize that these outputs are not substitutes but occur in fixed ratio. That is, for a fixed amount of input, an efficient transit system cannot increase vehicle miles by decreasing passenger miles, or increase passenger miles by decreasing vehicle miles. If an efficient transit agency is able to become even more efficient and increase vehicle miles while holding inputs fixed, then passenger miles also will increase if service elasticity is positive.

Real-world data are unlikely to present a constant ratio between vehicle miles and passenger miles because of random errors, variations in the efficiency with which each DMU produces its outputs (as, for example, if a DMU produces one output with 80 percent efficiency UIC Great Cities Institute and another output with 40 percent efficiency), and environmental factors such as per-capita income, population density, and fare differences. That is, holding inputs constant, the system that yields the most vehicle miles may not be the system that yields the most passenger miles, although there should be a strong positive correlation between the variables if they are nonsubstitutable, and a strong negative correlation between the variables if they are substitutes.

To test our hypothesis, we consider a sample of 55 United States bus transit operations from agencies with 150 or more buses in maximum service, with four observations (data for the years 2002-2005) for each agency (United States Federal Transit Administration, 2007). We have sampled only relatively large systems because only they are required to report the data necessary to compute input price differences. Also, because many of the agencies did not report complete data for each of the four years, we have limited the sample to those that did.

The sole input is operating costs adjusted for input price differences, and the two outputs are vehicle miles and passenger miles. By regressing one output on the other, while holding constant both the input and the three environmental influences noted above, we identify the relationship between the two outputs.

We can analyze these data with panel data analysis (PDA) because we have four years of data available. We estimated the unobserved effects with a fixed effect model because we are interested in the specific systems involved. We used robust error computation in order to correct for heteroskedasticity in the variances of the DMUs, which, not surprisingly, is present.

We empirically estimate the nature of the relationship between the two outputs by regressing the log of passenger miles on the log of vehicle miles, explicitly holding constant the log of adjusted operating expenses and the log of mean fare. The other two environmental influences, density and per-capita income, will be relatively constant for each DMU over the four-year period and therefore are included in the DMU-effect nuisance parameter. If the two

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outputs are nonsubstitutable, then the regression coefficient for the log of vehicle miles will be positive, being less than 1 if service elasticity shows decreasing returns to scale, exactly 1 for constant returns to scale, and greater than 1 for increasing returns to scale.

As the results show (Table 3), there is a strong positive relationship between passenger miles and vehicle miles, holding inputs and environmental influences constant. Note that service elasticity is estimated to be 0.74. This is about what we would expect, given that Litman's (2004) survey of previous studies reports that service elasticity varied between 0.6 and 1.0.

Table 3. Regression of Passenger Miles on Vehicle Miles: Fixed Effects Model, Urban Bus Systems

Robust Ln (Passenger Miles) Coef. Std. Err. t P>|t| Ln (Vehicle Miles) 0.740 0.097 7.62 0.000 Ln (Adjusted Expenses) 0.401 0.105 3.83 0.000 Ln (Fare) 0.080 2.46 0.015 -0.197 -1.043 0.791 1.31 Constant 0.192

DMU-Effect Variance: 0.0643. Idiosyncratic Variance: 0.0064 Fraction of variance due to differences among DMUs: 0.9091 (including differences in density and mean incomes)

Having shown empirically that the two outputs are nonsubstitutable, we illustrate the difference in DEA results for these data. We graph the two outputs for 2005 only, holding input constant. Because there is only one input and we are assuming constant returns to scale, we can divide each DMU's outputs by its input to standardize all DMUs; the resulting graph for 2005 data is shown in Figure 3. Both the conventional frontier and the fixed-proportion frontier are

the same except for the upper right corner of the graph, where the conventional frontier is shown with the dotted line and the fixed-proportion frontier remains the dashed line.



Figure 3. Urban Bus Passenger Miles per Dollar on Vehicle Miles per Dollar, ADD and FPA Frontiers

Next, we compute efficiency scores using the ADD model for the conventional frontier, and our FPA for the right angle frontier. Using the FPA model scores as the base, the DMUs are reported to be an average of 8.77 percent more efficient by the ADD model, with the range being from 1.48 percent to 100 percent.

### Application Involving Nonsubstitutable Inputs

Now, let us turn to inputs. We are able to illustrate the input case using a larger data set, because information about price differences across agencies is not needed. There are six years of data (1999-2004) for each of 67 American bus transit systems directly operated by agencies with 150 or more vehicles in maximum service. Again, all systems for which complete data are available are included.

Three inputs often used in transit DEA studies are maximum vehicles in service (a proxy for capital input), gallons of fuel (a proxy for energy input), and hours of operating labor (a proxy for labor input) (De Borger, Kerstens and Costa, 2002). We use these three as inputs in this study, and passenger miles as the one output measure.

We hypothesize that these three inputs are not substitutes but must be used in fixed proportions. Substitutability would mean that, for a given level of output, a transit agency could substitute labor for buses, buses for fuel, or fuel for labor. We argue that there is extremely limited substitutability in this industry; inputs have to be combined in virtually fixed ratios, with any excess wasted. (This reasoning has been confirmed by Joseph DiJohn, Research Professor, Urban Transportation Center, University of Illinois at Chicago. From 1983-1998, he was Chief Executive Officer of Pace, the Suburban Bus Division of the Regional Transportation Authority. Pace is responsible for all bus service in the six county region of Northeastern Illinois outside of the city of Chicago.)

If the inputs must be used in fixed proportions rather than being substitutable for each other, then holding output constant, the relationships between them will be positive. In order to identify the relationships, therefore, we need to conduct two regressions, in both cases holding output constant. We present the results in Tables 4 and 5.

Table 4.	Regression of Gallons of Fuel on Other Inputs,	Holding	Output	Constant,
	Fixed Effects Model, Urban Bus Systems			

I.n. (Gallons)	Coef	Robust Std. Err	t	P>ltl	
Ln (Maximum Vehicles)	0.213	0.057	3 740	0.000	
	0.210	0.007	0.740	0.000	
Ln (Operating Labor Hours)	0.296	0.060	4.940	0.000	
Ln (Passenger Miles)	0.205	0.037	5.560	0.000	
Constant	5.724	0.704	8.130	0.000	
DMU-Effect Variance:	0.0753	Idiosyncratio	variance:	0.0032	
Fraction of variance due to differences among DMUs: 0.9590					

Table 5.	Regression of Operating	Labor Hours of	on Other	Inputs,	Holding	Output	Constant,
	Fixed Effects Model,	Urban Bus Sy	/stems				

La (Oneration Labor Llaure)	Cast	Robust		
Ln (Operating Labor Hours)	Coer.	Std. Err.	t	P> t
Ln (Maximum Vehicles)	0.298	0.071	4.200	0.000
Ln (Gallons)	0.340	0.081	4.180	0.000
Ln (Passenger Miles)	0.170	0.044	3.890	0.000
Constant	4.471	1.089	4.100	0.000
DMU-Effect Variance:	0.0556	Idiosyncratio	c Variance:	0.0037
Fraction of variance due to d	ifferences a	mong DMUs:	0.9374	

As the tables demonstrate, there are statistically significant, positive relationships between all three of the inputs, confirming that they are used in fixed proportions. There certainly are no negative relationships, which would be present if the factors were substitutes.

Because it is difficult to graph the three dimensions clearly, we won't attempt to do so for the three inputs as we did for the two outputs. We computed efficiency scores for the 2004 data, with the following results. For the ADD model, there are three efficient DMUs. For the FPA model, none of the units are efficient, and all but the aforementioned three would have to decrease all three inputs to become efficient. For the aforementioned three, each would have to decrease two of their three inputs to become efficient. Using the FPA scores as the base, the DMUs are reported to be an average of 14.56 percent more efficient by the ADD scores, with the range being from 2.47 to 100 percent.

### CONCLUSIONS, CONJECTURES, AND RECOMMENDATIONS

Many recent DEA studies include nonsubstitutable inputs and/or outputs. If conventional DEA models are applied when some of the inputs are nonsubstitutable, or some of the outputs are nonsubstitutable, then technical efficiency scores will be biased upward.

The extent to which the scores are biased is inconsistent, so some DMUs will experience substantial bias while others will have little bias. For example, we applied both the conventional ADD model and our FPA model to real-world data that contained nonsubstitutable variables. Using the FPA model's scores as the base, the upward biases in efficiency estimates ranged from 1 to 100 percent in one data set and from 2 to 100 percent in the other.

On another topic, imposing weight restrictions is quite common in DEA studies, often because of the bizarre input and output weights that occur when there are no restrictions. It has long seemed to us that if the assumptions underlying the DEA model being used were correct, then the unrestricted weights should conform to reasonable marginal rates of substitution for inputs and marginal rates of transformation for outputs.

Subsequent to the completion of the research reported herein, we surmised that one of the reasons for the unrestricted weight problems might be that the conventional DEA models assumed substitutable variables when the variables actually were nonsubstitutable. To test this conjecture, we ran a simple simulation of 500 random observations using a Cobb-Douglas function with two inputs and one output and constant returns to scale, with 200 of the observations being efficient, and the remainder inefficient in one or both inputs. For one simulation we chose both inputs randomly (so the inputs were substitutable), and for the other we chose the first input randomly and used the same random number for the second input (so the inputs were nonsubstitutable). We used the Charnes-Cooper-Rhodes (CCR) radial model (Charnes *et al.*, 1994) to compute DEA scores.

When the two inputs were substitutable, in almost all of the cases the two inputs both were given reasonable weights and there were few slacks. When the two inputs were nonsubstitutable, every DMU put all of the weight on only one of its inputs, and many DMUs showed substantial slacks. Although we cannot draw strong conclusions, it seems to us that a

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key reason for weird weights and large slacks may be related to using DEA models that assume variable substitutability when, in fact, the variables are nonsubstitutable.

There are several ways to deal with the problem of nonsubstitutable variables. The first is to develop and use DEA models that treat fixed-ratio inputs (outputs) as nonsubstitutable variables rather than as substitutable variables, as we have demonstrated herein using the FPA model. We have specifically avoided complex situations and formal mathematical DEA models in this paper, to insure that the focus remains on the basic underlying problem. In the future, mathematical models need to be developed for situations where there are multiple inputs and multiple outputs, where both inputs and outputs include nonsubstitutable variables, and where some inputs (outputs) are nonsubstitutable and other inputs (outputs) are substitutable.

A second solution is to use conventional DEA models, but aggregate nonsubstitutable variables using their prices as weights. We believe that this solution is a good one if prices are available and can be adjusted for price differences over time and among DMUs. These models also deal with the problem of a DMU being more efficient in its production of one type of output than another, or more efficient in the use of one type of input than another. This solution is the one we used in our first application – we aggregated nonsubstitutable physical inputs by using operating expenses as the sole input variable.

A third solution is to use conventional DEA models, but utilize only one of the nonsubstitutable inputs and only one of the nonsubstitutable outputs. Because fixed-ratio variables are nonsubstitutable, they will increase and decrease together, so one can serve as a rough proxy for all. The problem with this approach is that it does not account for differences in a DMU's efficiency in producing different outputs or in using different inputs. But, in the absence of comparable prices, it may be the best choice available if one wishes to use

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conventional DEA models. This is the solution that we used in our second application – we used passenger miles as the sole output, omitting the output of vehicle miles.

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