Optimal Leverage in Real Estate Investment with Mezzanine Lending

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Executive Summary

The paper presents a theoretical analysis of the optimal leverage for the purpose of investing in real estate under the condition that borrowing in excess of a standard amount such as 70 to 80 percent of the purchase price must be accomplished through a mezzanine loan with a high rate of interest. The conditions under which a mezzanine loan is used are derived. It is shown that a larger mezzanine loan is used the greater is the required expected after-tax rate of return to equity. Investors who choose greater risk require a higher expected after-tax return to equity and therefore borrow more and purchase more real estate with a given equity investment.

Introduction

Many real estate investors borrow a large fraction of the purchase price of the properties in which they invest. A real estate investment is often financed by a "capital stack" that consists of the permanent loan of 70 to 80 percent of the purchase price, the equity investment, and a mezzanine loan that fills the gap between the permanent loan and the equity investment. The equity investment may consist of the investor's own equity plus an equity position that is taken by an equity financer such as a mortgage banker or investment banking company. Bergsman (2006) provides a good description of the various forms of the capital stack currently in use. This paper examines the question of the optimal leverage (percentage borrowed) when mezzanine lending is available at what is usually a high rate of interest. A larger percentage borrowed means that the investor can spread his/her equity over a larger amount of property investments, thereby taking on more risk and achieving a higher expected rate of return to equity. The basic result of the paper is that investors may use mezzanine financing above the permanent loan even if the interest rate on the mezzanine loan exceeds the target after-tax rate of return to equity. It is the explanation of this seemingly counter-intuitive result with which this paper is concerned. Optimal leverage in real estate investment has been studied previously by Cannaday and Yang (1995, 1996), Gau and Wang (1990), and McDonald (1999), but their models do not include mezzanine loans (although they do include the assumption that the interest rate increases with the percentage borrowed).

As an introductory example, suppose that the investor has $1 million to invest in real estate, and that conventional loans are available to finance 75 percent of the purchase price(s). This means that an investor can purchase real estate worth $4 million with the equity of $1 million. That real estate may be a single property, or a combination of two or more properties that add up to $4 million. However, suppose now that mezzanine financing is also available, and that the investor finances 15 percent of the total real estate investment from this source. The investor now has borrowed 90 percent of the purchase price of the investment properties, and therefore has purchased real estate in the amount of \$10 million using a \$1 million equity investment. The total value of the real estate investment is highly sensitive to the degree of leverage in this range.

The Basic Model

Consider a simplified model of a real estate investor who invests equity amount EI by purchasing property at the beginning of the time period and sells it at the end of the time period. The question is how much to borrow – and therefore how much to spend on real estate investments, given the amount of equity at the investor's disposal. The model assumes that the menu of choices for the investor follows the single-period capital asset pricing model and therefore consists of the points on the security market line, which can be written:

$$E(r_i) = r_f + \beta_i[E(r_m) - r_f]. \qquad (1)$$

$E(r_i)$ is the expected rate of return, r_f is the risk-free rate of return, $E(r_m)$ is the expected rate of return to the entire market basket of investments, and $\beta_i = \mathrm{cov}(r_i, r_m)/\sigma_m^2$. See Briedenbach, Mueller, and Schulte (2006) for estimates of real estate "betas."

The quantity $[E(r_m) - r_f]/\sigma_m^2$ is the same for all investments and is known as the price of risk. Equation (1), the security market line, can be rewritten as

$$E(r_i) = r_f + (\text{price of risk}) * \mathrm{cov}(r_i, r_m). \qquad (1')$$

Now interpret equation (1') as the equation for the expected return to equity invested in real estate. Purchasing more property with a given equity investment (i.e., borrowing more) is the procedure used to increase the covariance between the real estate investment(s) and the market return, assume more risk, and increase the expected return to equity. The crucial assumption is that the overall interest rate on borrowed funds increases with the percentage of the purchase price borrowed if a mezzanine loan is used.

The net present value of equity (NPVE) is:

$$NPVE = [ATCF/(1+y)] + [ATER/(1+y)] - EI, \qquad (2)$$

where

ATCF = after-tax cash flow,

ATER = after-tax equity reversion,

EI = equity investment, and

y = required expected after-tax return to equity.

The details of the model are as follows. The after-tax cash flow is the net operating income after tax plus the cash value of the depreciation allowance minus the after-tax cost of borrowing:

$$ATCF = (1-t)NOI + tDV - (1-t)imV, \qquad (3)$$

where

NOI = net operating income,

t = income tax rate,

D = depreciation rate for income tax purposes,

V = purchase price of property investments,

i = interest rate on loan, and

m = proportion of purchase price borrowed.

The after-tax equity reversion is the net selling price minus the remaining balance on the loan and the tax on the capital gain:

$$ATER = NSP - mV - t[NSP - V + DV], \qquad (4)$$

where

NSP = net selling price of the properties (after transactions costs).

The equity investment is related to the total value of property investments as

$$EI = (1\text{-}m)V, \qquad (5)$$

so V/EI = 1/(1-m). As noted above, the degree of leverage is 4 when m = 0.75, and is 10 when m = 0.90. The degree of leverage is highly sensitive to the percentage borrowed in this range.

The overall interest rate on borrowed funds is the weighted average of the interest rates on the permanent loan and the mezzanine loan;

$$i = [i_0 L + (m - L)i_\mu]/m, \quad \text{for } 1.00 \geq m \geq L. \qquad (6)$$

where

i_0 = interest rate on the permanent loan,

i_μ = interest rate on the mezzanine loan, and

L = the limit on the permanent loan (e.g., 75 percent, written as 0.75).

Note the assumption that the borrower cannot borrow more than 100 percent of the purchase price, so 1.00 ≥ m. We assume for simplicity that the interest rate on borrowed funds increases with the percentage borrowed above the limit of the permanent loan:

$$i = i_0 + b(m - L), \qquad \text{for } 1.00 \geq m \geq L, \qquad (7)$$

where b is the increase in the overall interest rate as the percentage borrowed rises above L. The assumption of a linear function for the overall interest rate requires that the rate on the mezzanine loan vary with m. In particular, it is shown in the appendix that the interest rate on the mezzanine loan is

$$i_\mu = i_0 + bm. \qquad (8)$$

The interest rate that the investor faces jumps from i_0 to i_0 + bL at L, the limit of the standard permanent loan. If L = 0.75 and b = 0.1, for example, then the interest rate jumps by 0.075 at L, and then increases with m above L at rate b. The parameter b determines both the size of the jump and the additional increase as m increases above L. The implications of this interest rate schedule are explored below. The use of this particular functional form for the overall interest rate (i) simplifies the computation of the model.

Analysis of the Model

The purpose of the analysis is to find the optimal amount to borrow, given the interest rates, the required expected after-tax rate of return, and the tax rate. Rewrite the basic model as

$$(1+y)NPVE = ATCF + ATER - EI(1+y). \quad (9)$$

Differentiate with respect to m and set equal to zero as follows:

$$(1+y)[\partial NPVE/\partial m] = - (1-t)iV - (1-t)mV(di/dm) - V + (1+y)V = 0. \quad (10)$$

Cancel out V, and the solution for optimal m is

$$m^* = [y - (1-t)i]/(1-t)(di/dm). \quad (11)$$

The denominator of the expression is greater than zero, so the condition for any borrowing (m > 0) is that the expected after-tax return to equity exceeds the overall after-tax cost of borrowing; i.e., [y – (1-t)i] is greater than zero. It can be shown readily that these results hold for this model:

- An increase in y, the required expected after-tax return to equity, increases m* and there for increases V, the total amount of property purchased.
- An exogenous increase in i, the overall interest rate, reduces m* and V.
- An increase in di/dm reduces m* and V.
- An increase in t, the tax rate, increases m* and V.

The first three results are obvious by inspection of equation (11). A demonstration of the fourth result – the effect of an increase in the tax rate – is not as obvious. Use of the quotient rule for derivatives yields

$$dm^*/dt = y(di/dm)/[(1-t)di/dm]^2. \qquad (12)$$

The derivative dm*/dt is greater than zero because di/dm is greater than zero by assumption.

Equation (11) is the general solution for the model, but insertion of equation (6) – the equation for the overall interest rate – into equation (10), produces an illuminating result that serves as a basis for numerical examples. Substituting $i = i_0 + b(m - L)$ and $di/dm = b$ produces

$$m = [y - i_0(1-t) + bL(1-t)]/2b(1-t). \qquad (13)$$

As an example, assume the following parameter values:

$i_0 = 0.08$

$t = 0.30$,

$L = 0.75$

$b = 0.10$ [$i_\mu = 0.18$ if $m = 1.00$]

The assumption that b = 0.10 is based on the idea that the overall interest is 8% with no mezzanine loan (m = 0.75) and is 10.5% if the investor borrows 100 percent of the purchase price. In this case the interest rate on the mezzanine loan of 25 percent of the purchase price is 18%. Insertion of these parameter values into equation (13) produces

$$m^* = [y - 0.056 + 0.0525]/0.2(0.7) = [y - 0.0035]/0.14. \qquad (14)$$

Exhibit 1 relates m* and the required expected after-tax return y, the overall interest rate, and the mezzanine interest rate.

<< Exhibit 1 here >>

The investor who wishes to assume more risk increases the required expected after-tax return to equity and borrows more. Note in this case that, while the interest rate on the mezzanine loan is higher, the after-tax interest rate on the mezzanine loan is lower than the required expected after-tax return to equity. The intuition of the basic result is that, if the investor desires to assume more risk and increase the required expected after-tax return to equity, then he/she is willing to borrow more as long as the increase in the return to equity is greater than the increase in the borrowing rate.

The numerical results are sensitive to the rate at which the overall cost of borrowing increases with m. If b = 0.15 instead of 0.10 (with no change in the other parameters), the equation for the optimal borrowing percentage is

$$m^* = [y + 0.02275]/0.21 \tag{15}$$

In this case the investor begins to access mezzanine financing (m* = 0.75) at a required expected after-tax return of 13.475 percent, and seeks to borrow 100 percent of the purchase price at a required expected after-tax return of 18.725 percent. The overall interest rate is 11.75 percent with a mezzanine loan of 25 percent of the purchase price.

Conclusion

This short note has explored the economics of maximizing the net present value of the equity of a real estate investor by engaging in leverage. The crucial feature of the model is the high interest rate charged for mezzanine loans, which are loans in excess of a standard percentage of the price of the investment properties. Standard terms for real estate loans limit the borrower to 70 percent to 80 percent of the purchase price, but mezzanine loans are available at much higher rates of interest. Nevertheless, investors have an incentive to use mezzanine lending under certain conditions. In one example, an investor with a required expected after-tax return to equity of 14.35 percent who faces an interest rate of 8.0 percent on the standard 75 percent permanent loan and has a tax rate of 30 percent, is willing to borrow a mezzanine loan at 18.0 percent interest up to 100 percent of the purchase price. Note that the after-tax borrowing rate on this mezzanine loan is 12.6 percent. In contrast, an investor in the same situation with a required expected after-tax return to equity of 12 percent borrows 83 percent of the purchase price.

Several extensions of the model can be pursued. The number of time periods can be made arbitrarily large and additional options for financing can be added (e.g., equity investors with various requirements). Loans can be made more complex with the addition of discount points, prepayment penalties, recourse versus nonrecourse features, and so on. The purpose of this paper is the present a reasonably simple analytical treatment of optimal leverage that can point the way to more complex models which will probably need to be formulated as computer programs.

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Appendix

This appendix investigates the relationship between the overall rate of interest and the interest rate on the mezzanine loan. The general form for the overall rate of interest is

$$i = i_0(L/m) + i_\mu(m - L)/m. \qquad \text{(A1)}$$

The interest rate on the mezzanine loan can therefore be written

$$i_\mu = [i - i_0(L/m)][m/(m - L)]. \qquad \text{(A2)}$$

It is assumed that $i = i_0 + b(m - L)$, so

$$i_\mu = [i_0 + b(m - L) - i_0(L/m)][m/(m-L)]$$

$$= i_0 + bm. \qquad \text{(A3)}$$

Equation (A3) says that the interest rate on the mezzanine loan in the model starts at $i_0 + bL$, which for example is 0.08 + 0.1x0.75 = 0.155 at L = 0.75. This interest rate then increases by b as m increases above 0.75. For example, $i_\mu = 0.17$ at m = .90 and 0.18 at m = 1.00.

Exhibit 1
Expected After-Tax Return to Equity and Optimal Percentage Borrowed

Expected after-tax return to equity (y)	Optimal percentage borrowed (m*)	Overall interest rate (i)	Interest rate on mezzanine loan (i_μ)	$(1-t)i_\mu$
0.1085 or less	0.75	0.08	0.155	0.1085
0.11	0.76	0.081	0.156	0.109
0.12	0.83	0.088	0.163	0.114
0.13	0.90	0.095	0.170	0.119
0.14	0.97	0.102	0.177	0.124
0.1435 or over	1.00	0.105	0.189	0.126

Note: Table is based on conventional loan of 75 percent at 8 percent interest, tax rate of 30 percent, and b=0.1.